Paper

A study of a Fuchsian system of rank 8 in 3 variables and the ordinary differential equations as its restrictions, by Akihito Ebisu, Yoshishige Haraoka, Masanobu Kaneko, Hiroyuki Ochiai, Takeshi Sasaki and Masaaki Yoshida, to appear in Osaka Journal of Mathematics

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Explanation of data

The following data are included:

- Equation $Z_3(A)$ is given in Section 1 as a system of differential equations $\{E_1, E_2, E_3\}$ with parameters a_0, a_1, a_2 and a_3. The parameters A_i are introduced by the relations

$$a_0 = 2A_0,\ a_i = A_i^2 - (A_0 - 1)^2 \quad i = 1, 2, 3.$$

  The pfaffian form ω defined in Subsection 1.1 as $de = \omega e$ is given as the 8×8-matrix M whose components are written as $M[i, j]$, $1 \leq i, j \leq 8$. They are saved in the file **Z3Mmatrix.txt**, where dti denotes the 1-form dt_i.

- The ordinary differential equation of rank 8 denoted as $Z_{\Delta 8}(A)$ is written as

$$z8 = C0 * z0 + C1 * z1 + C2 * z2 + C3 * z3 + C4 * z4 + C5 * z5 + C6 * z6 + C7 * z7,$$

  where $zi = d^i z/dt^i$. Let F be the least common multiple of the denominators of the coefficients Ci; then, it is expressed as

$$F = 64(2t + 1)^{13}(t - 1)^{13}(t + 1)^{24}(t + 2)^{17}Dt.$$

  The coefficients and Dt are saved in the file **ode8mpl.txt**.

- The ordinary differential equation of rank 6 denoted as $Z_{\Delta 6}(A)$ is written as

$$z6 + D5 * z5 + D4 * z4 + D3 * z3 + D2 * z2 + D1 * z1 + D0 * z0 = 0,$$

  where the coefficients Di are saved in the file **ode6mpl.txt**. We use the parameters a_0, a_1, and $p4$, where $a_2 = a_3 = p$.

- The ordinary differential equation of rank 4 denoted as $Z_{\Delta 4}(A)$ is written as

$$z4 + E3 * z3 + E2 * z2 + E1 * z1 + E0 * z0 = 0,$$

  where the coefficients Ei are saved in the file **ode4mpl.txt**. We use the parameters a_0, $p = a_1 = a_2 = a_3$.

- Let $z(t_1, t_2, t_3)$ be any solution of $Z_3(A)$. If it is regarded as a function only of t_1, it satisfies an ordinary differential equation of rank 8 as

$$P8 * z8 + P7 * z7 + P6 * z6 + P5 * z5 + P4 * z4 + P3 * z3 + P2 * z2 + P1 * z1 + P0 * z0 = 0$$

  where $zi = d^i z/dt^i$, $t = t_1$, which we call the section of $Z_3(A)$ relative to t_1. The coefficients $P8$ is of the form

$$(t + 1)^2(t - 1)^2(1 - t^2 - t_2^2 - t_3^2 + 2t_2t_3t)^4 P(t)$$

  for a polynomial $P(t)$ of degree 16. Concrete representation of coefficients is not easy and we give in **Z3sectiondata.txt** the coefficients Pi when $t_2 = 5$ and $t_3 = 3$.

- Equation $Z_2(A)$ given in Section 3 is written $de_6 = \omega_6 e_6$, where the Pfaffian form ω_6 is a 6×6-matrix 1-forms $N_1 dt_1 + N_2 dt_2$. The matrices N_1 and N_2 are saved in the file **Z2Nmatrix.txt**.

- Any solution $z(t_1, t_2)$ of $Z_2(A)$ regarded as a function of $t = t_1$ satisfies an ordinary differential equation of the form

  $$Q6 * z6 + Q5 * z5 + Q4 * z4 + Q3 * z3 + Q2 * z2 + Q1 * z1 + Q0 * z0 = 0$$

  where $zi = d^i z/dt^i$, $t = t_1$, which we call the section of $Z_2(A)$ relative to t_1. The coefficients $Q6$ is of the form

  $$(t+1)^2(t-1)^2(t-t_2)^4 Q(t)$$

  for a polynomial $Q(t)$ of degree 6. The coefficients Qi are given in **Z2sectiondata.txt**.