Risa/Asir

1.0
2004 8

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1

1.1

- 2
-
- neighborhood graph tree
-
- (adjoint curves)
-
-

  x,y,z

1.2 Notation

- `[x,y,z]` *(x:y:z)z=0 z=1*
- Q $\overline{Q}$

1.3

1.3.1 `intersect`

`intersect(`*F*`,`*G*`)`
:: *2F=0,G=0* .

return

F G x,y,z

- *2F=0,G=0* `[x,y,z]`
- *F,G*

```
[1] intersect(y^2-x*z,(x^2+y^2)^3-4*x^2*y^2*z^2);
[[0,0,1],[(#4),(#5),1]]
[2] defpoly(alg(4));
t#4^3+3*t#4^2+3*t#4-3
[3] defpoly(alg(5));
t#5^2-t#4
[4] intersect(x^2-y^2,x^3+y*x^2+(y^2-z^2)*x+y^3-z^2*y);
***two curve have common components***
```

1.3.2 `sing`

`sing(`*F*`)` :: *F=0* .

return

F x,y,z

- *F=0* `[x,y,z]` $F_x(x,y,z) = F_y(x,y,z) = F_z(x,y,z) = 0$
- *F*

```
[1] sing(16*x^6-24*z^2*x^4+9*z^4*x^2+4*z^2*y^4-4*z^4*y^2);
[[0,0,1],[(#4),0,1],[1/2,(#3),1],[-1/2,(#3),1],[0,1,0]]
[2] defpoly(alg(3));
2*t#3^2-1
[3] defpoly(alg(4));
4*t#4^2-3
[4] sing((x-y)*(y^2-x*z));
[[1,1,1],[0,0,1]]
[5] sing((x-y)^2*(y^2-x*z));
***Argument has multiple divisor***
```

Section 1.3.3 [nbh], page 2, Section 1.4.4 [multia], page 5,

1.3.3 nbh

`nbh(F)` :: *F*=0 neighborhood graph

return

F x,y,z

- *F=0* neigborhood graph neighborhood graph tree

  [, , [, (=1)(=-1)], [("terminal")]]

  [, [, (=1)(=-1)], [("terminal")]]

  `=1`

- neighborhood graph

```
[1] F=x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6;
x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6
[2] sing(F);
[[0,0,1],[(#0),1,0]]
[3] nbh(F);
[ 1 [0,0,1] [4,-1] [[ 1 [2,1] [terminal] ],[ 1 [2,1] [terminal] ]] ]
[ 2 [(#0),1,0] [2,-1] [[ 1 [1,1] [terminal] ]] ]
```

`[0,0,1]` 4 22 `[(#0),1,0]`

- *F*

Section 1.3.2 [sing], page 1,

1.3.4 genus

`genus(F)` :: *F=0* .

return 0

F x,y,z

- *F=0*

- F $\overline{Q}[x,y,z]$ Q[x,y,z] $\overline{Q}[x,y,z]$

```
[1] genus(x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6);
0
[2] genus(y^2*z-x^3-z^3);
1
[3] genus(x^2+y^2+z^2-x*y-y*z-z*x);
-1
[4] fctr(x^2+y^2+z^2-x*y-y*z-z*x);
[[1,1],[x^2+(-y-z)*x+y^2-z*y+z^2,1]]
[5] irr_conic(x^2+y^2+z^2-x*y-y*z-z*x);
reducible
```

Section 1.4.5 [irr_conic], page 6,

1.3.5 adjoint1,adjoint2

`adjoint1(`*F*`)`
`adjoint2(`*F*`)`
:: *F=0*n-1,n-2(adjoint curve)(n=deg(F))

return x,y,z

F x,y,z

- n-2 *G=0F=0* r r-1 *G=0F=0* n-2 (adjoint curve)n-1 $G_0 = 0, G_1 = 0, \ldots, G_{n-2} = 0$ n-2 $c_0G_0 + c_1G_1 + \ldots + c_{n-2}G_{n-2}$ (c_i) `adjoint2(`F`)` n-1 n-1 n-1 2n-1 n-1 `adjoint1(`F`)`
-
- *F*

```
[1] adjoint2(x^6+3*y^2*x^4+(3*y^4-4*z^2*y^2)*x^2+y^6);
[c2,c3,c4,c6,c7] 5
(c2-c4)*x^4+c3*y*x^3+(c2*y^2+c6*z*y)*x^2+(c3*y^3+c7*z*y^2)*x+c4*y^4
[2] adjoint1(F);
[c1,c7,c11,c12,c13,c15,c16,c17,c18,c19,c20] 11
(c1*y+(c11-c15+c18-c20)*z)*x^4+(c13*y^2+c7*z*y+c11*z^2)*x^3+(c17*z*y^2+c12*z^2*y
+c15*z^3)*x^2+(c13*z^2*y^2+c16*z^3*y+c18*z^4)*x+c17*z^3*y^2+c19*z^4*y+c20*z^5
```

Section 1.4.7 [restriction], page 6,

1.3.6 intpt

`intpt(`*F*`)` :: *F=0* `[x,y,z] no integer solution`

return `no integer solution`.

F x,y,z

- *F=0* (affine) `[x,y,z]`x,y, `z no integer solution`
- Legendre*F*

```
[1] intpt(22*x^2-10*y^2+z^2+5*x*y+13*y*x-z*x);
[71,-121,473]
[2] intpt(22*x^2-10*y^2+z^2+5*x*y+12*y*x-z*x);
no integer solution
```

1.3.7 parametrize

`parametrize(F)`
:: *F=0*

return

F x,y,z

- *F=0*0t P(t),Q(t),R(t) x,y,zS(x,y,z),T(x,y,z)(x:y:z)=(P(t):Q(t):R(t)), t=T(x,y,z)/S(x,y,z) `parametrize`(*F*) `[P(t),Q(t),R(t),T(x,y,z)/S(x,y,z)]` GCD(`P(t)`,`Q(t)`,`R(t)`)=1 P(t),Q(t),R(t)
- *F* $\overline{Q}[x, y, z]$ 0

```
[1] parametrize(x^4+(2*y^2-z^2)*x^2+y^4+z^2*y^2);
[-t^3-t,t^3-t,t^4+1,(-x^2-y^2)/(z*x+z*y)]
[2] parametrize((x^2+y^2)^3-4*x^2*y^2*z^2);
heuristic2 failed...
heuristic3 succeed
[32256*t^6-133120*t^5-129024*t^4+1064960*t^3-516096*t^2
-2129920*t+2064384,-127008*t^6+1048320*t^5-2671232*t^4
+10684928*t^2-16773120*t+8128512,274625*t^6-3194100*t^5
+15678780*t^4-41555808*t^3+62715120*t^2-51105600*t+17576000,
(-126*x^4+1040*y*x^3-382*y^2*x^2+1040*y^3*x-256*y^4)
/(-65*x^4+520*y*x^3+(-65*y^2-32*z*y)*x^2+(520*y^3+256*z*y^2)*x)]
[3] parametrize(22*x^2-10*y^2+z^2+5*x*y+12*y*x-z*x);
[(220*#6-10)*t^2+(-22*#6+1),(374*#6-17)*t^2+(-22*#6-43)*t,
(220*#6+210)*t^2+(-374*#6+17)*t+22,(-y)/((22*#6-1)*x+z)]
```

Section 1.3.4 [genus], page 2,

1.4

1.4.1 tdeg

`tdeg(Poly)`
:: *Poly*

return 0

Poly

- *Poly*

```
[1] tdeg(u^3+v^3-x*y*z*w);
4
[956] tdeg((x^3+y^2+z)*(a^2+b+1));
5
```

1.4.2 homzation

`homzation(AF)`
:: x,yx,y,z

return x,y,z

F x,y

- x,yx,y,zx,y

```
[1] homzation((x^2+4*x^3+6*x^4)-4*x^4*y
+(-2*x-4*x^2-2*x^3)*y^2+y^4);
(-4*y+6*z)*x^4+(-2*y^2+4*z^2)*x^3
+(-4*z*y^2+z^3)*x^2-2*z^2*y^2*x+z*y^4
[958] homzation(u*v+1);
Input must be polynomial of variable x,y
```

1.4.3 random_line

`random_line(`*Pt*`,B[,`*Seed*`])`
:: *Pt*(=`[x,y,z]`)

return x,y,z

Pt

B

Seed

- *Pt*(=`[x,y,z]`) -BB
- Seedrandom([Seed])

```
[1] random_line([0,0,1],1);
x-8*y
```

1.4.4 multia

`multia(F,Pt)`
:: *F=0* *Pt*(=`[x,y,z]`)

return 0

F x,y,z

Pt

- *F=0* *Pt*(=`[x,y,z]`) FN Pt 0N*F=0*Pt

```
[1] multia((4*y^2+4*z^2)*x^4+8*z^3*x^3+8*z^2*y^2*x^2-8*z^5*x+
4*z^4*y^2-4*z^6,[0,0,1]);
0
[2] multia((4*y^2+4*z^2)*x^4+8*z^3*x^3+8*z^2*y^2*x^2-8*z^5*x+
4*z^4*y^2-4*z^6,[0,1,0]);
4
[3] multia((4*y^2+4*z^2)*x^4+8*z^3*x^3+8*z^2*y^2*x^2-8*z^5*x+
4*z^4*y^2-4*z^6,[1,0,0]);
2
```

Section 1.3.2 [sing], page 1, Section 1.3.3 [nbh], page 2,

1.4.5 irr_conic

`irr_conic(F)`
:: F $\overline{Q}[x,y,z]$

return

F x,y,z

- F $\overline{Q}[x,y,z]$ `irreduciblereducible`

```
[1] irr_conic(x^2+y^2+z^2-x*y-y*z-z*x);
reducible
[2] fctr(x^2+y^2+z^2-x*y-y*z-z*x);
[[1,1],[x^2+(-y-z)*x+y^2-z*y+z^2,1]]
```

1.4.6 lissajou

`lissajou(M,N)`
:: $x = \sin(M\theta), y = \cos(N\theta)$

return x,y,z

M N

- $x = \sin(M\theta), y = \cos(N\theta)$ x,y,z

```
[984] lissajou(3,4);
64*x^8-128*z^2*x^6+80*z^4*x^4-16*z^6*x^2+16*z^2*y^6
-24*z^4*y^4+9*z^6*y^2
[985] lissajou(2,7);
4096*x^14-14336*z^2*x^12+19712*z^4*x^10-13440*z^6*x^8
+4704*z^8*x^6-784*z^10*x^4+49*z^12*x^2+4*z^10*y^4-4*z^12*y^2
```

1.4.7 restriction

`restriction(A,List)`
::

return x,y,z

A `adjoint1,adjoint2` x,y,z

List `[x,y,z]`

- `adjoint1,adjoint2` *List* Q
- *List*`intersectsing`

⟨undefined⟩ [adjoint1], page ⟨undefined⟩,

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