

# **nn\_ndbf**

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nn\_ndbf User's Manual  
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In this manual we explain about a new b-function package ‘`nn_ndbf.rr`’ in `asir-contrib`. To use this package one has to load ‘`nn_ndbf.rr`’.

```
[...] load("nn_ndbf.rr");
```

A prefix `ndbf.` is necessary to call the functions in this package. In this manual we also explain about some related built-in functions.

## 0.1 Computation of b-function

### 0.1.1 `ndbf.bfunction`

`ndbf.bfunction(f[|weight=w,heruristic=yesno,vord=v,op=yesno])` :: computes the global b-function of a polynomial  $f$

*return* a polynomial

$f$  a polynomial

$w$  a list  $[w_1, w_2, \dots, w_n]$

*yesno* 0 or 1

$v$  a list of variables

- This function is defined in an `asir-contrib` package ‘`nn_ndbf.rr`’.
- This function computes the global b-function of a polynomial  $f$ . By default only the global b-function is returned. If an option `op=1` is given, a pair  $[b, P]$  of the global b-function and a differential operator satisfying  $Pf(s+1)=b(s)f^s$ . The operator  $P$  is represented as a commutative polynomial of variables  $v_1, \dots, v_n, dv_1, \dots, dv_n$ . The d-variables are treated as commutative indeterminates in this representation and the polynomial should be regarded as a canonical representation with each polynomial coefficient placed at the left of d-variables.
- If an option `weight=[w1, w2, ..., wn]` is given, the computation is done with a weight  $(w_1, \dots, w_n)$  for  $(v_1, \dots, v_n)$ . This option is useful when  $f$  is weighted homogeneous with respect to  $(w_1, \dots, w_n)$ .
- If an option `heuristic=1` is given a change of ordering is done before entering elimination. In some cases this improves the total efficiency.
- The variable order used in the whole computation is automatically set by default. If an option `vord=v` is given, a variable order  $v$  is used instead.

```
[...] load("nn_ndbf.rr");
[...] ndbf.bfunction(x^3-y^2*z^2);
-11664*s^7-93312*s^6-316872*s^5-592272*s^4-658233*s^3-435060*s^2
-158375*s-24500
[...] ndbf.bfunction(x^3-y^2*z^2|op=1);
[-11664*s^7-93312*s^6-316872*s^5-592272*s^4-658233*s^3-435060*s^2
-158375*s-24500, (108*z^3*x*dz^3+756*z^2*x*dz^2+1080*z*x*dz+216*x)*dx^4
...
+(729/8*z^3*dz^5+9477/8*z^2*dz^4+5103/2*z*dz^3+2025/2*dz^2)*dy^2]
[...] F=256*u1^3-128*u3^2*u1^2+(144*u3*u2^2+16*u3^4)*u1-27*u2^4
-4*u3^3*u2^2$
```

```
[...] ndbf.bfunction(F|weight=[u3,2,u2,3,u1,4]);
576*s^6+3456*s^5+8588*s^4+11312*s^3+8329*s^2+3250*s+525
```

### 0.1.2 ndbf.bf\_local

`ndbf.bf_local(f,p[|weight=w,heruristic=yesno,vord=v,op=yesno])` :: computes the local b-function of a polynomial  $f$  at  $p$ .

*return* a list

$f$  a polynomial

$p$  a list  $[v1, a1, \dots, vn, an]$

$w$  a list  $[w1, w1, \dots, vn, wn]$

*yesno* 0 or 1

$v$  a list of variables

- This function is defined in an asir-contrib package ‘`nn_ndbf.rr`’.
- This function computes the local b-function of a polynomial  $f$  at a point  $(v1, \dots, vn) = (a1, \dots, an)$ . The output is a list of pairs of each factor of the local b-function and its multiplicity.
- By default only the local b-function is returned. If an option `op=1` is given, a triple  $[b, a, P]$  of the local b-function, a polynomial and a differential operator satisfying  $P\hat{f}(s+1) = ab(s)\hat{f}^s$ . The operator  $P$  is represented as a commutative polynomial of variables  $v1, \dots, vn, dv1, \dots, dvn$ . The d-variables are treated as commutative indeterminates in this representation, the polynomial should be regarded as a canonical representation with each polynomial coefficient placed at the left of d-variables.
- If an option `weight=[w1, w1, \dots, vn, wn]` is given, the computation is done with a weight  $(w1, \dots, wn)$  for  $(v1, \dots, vn)$ . This option is useful when  $f$  is weighted homogeneous with respect to  $(w1, \dots, wn)$ .
- If an option `heuristic=1` is given a change of ordering is done before entering elimination. In some cases this improves the total efficiency.
- The variable order used in the whole computation is automatically set by default. If an option `vord=v` is given, a variable order  $v$  is used instead.

```
[...] load("nn_ndbf.rr");
[...] ndbf.bf_local(y*((x+1)*x^3-y^2), [x, -1, y, 0]);
[[-s-1, 2]]
[...] ndbf.bf_local(y*((x+1)*x^3-y^2), [x, -1, y, 0] |op=1);
[[[-s-1, 2]], 12*x^3+36*y^2*x-36*y^2, (32*y*x^2+56*y*x)*dx^2
+((-8*x^3-2*x^2+(128*y^2-6)*x+112*y^2)*dy+288*y*x+(-240*s-128)*y)*dx
+(32*y*x^2-6*y*x+128*y^3-9*y)*dy^2+(32*x^2+6*s*x+640*y^2+39*s+30)*dy
+(-1152*s^2-3840*s-2688)*y]
```

### 0.1.3 ndbf.bf\_strat

`ndbf.bf_strat(f[|weight=w,heruristic=h,vord=v])`

:: computes a stratification associated with local b-function of a polynomial  $f$ .

*return* a list

*f* a polynomial  
*w* a list [*v1,w1,...,vn,wn*]  
*h* 0 or 1  
*v* list of variables

- This function is defined in an asir-contrib package ‘`nn_ndbf.rr`’.
- This function computes a stratification associated with local b-function of a polynomial *f*. The output is a list [*s1,...sl*] where each *si* is a list [*l1,l2,bi*]. In this list, *l1* and *l2* are generators of ideals and they represent the local b-function is *bi* over  $V(l1)-V(l2)$ .
- If an option `weight=[v1,w1,...,vn,wn]` is given, the computation is done with a weight (*w1,...,wn*) for (*v1,...,vn*). This option is useful when *f* is weighted homogeneous with respect to (*w1,...,wn*).
- If an option `heuristic=1` is given a change of ordering is done before entering elimination. In some cases this improves the total efficiency.
- The variable order used in the whole computation is automatically set by default. If an option `vord=v` is given, a variable order *v* is used instead.

```
[...] load("nn_ndbf.rr");
[...] F=256*u1^3-128*u3^2*u1^2+(144*u3*u2^2+16*u3^4)*u1-27*u2^4
-4*u3^3*u2^2$
[...] ndbf.bf_strat(F);
[[[u3^2,-u1,-u2],[-1],[[-s-1,2],[16*s^2+32*s+15,1],[36*s^2+72*s+35,1]]],
[[-4*u1+u3^2,-u2],[96*u1^2+40*u3^2*u1-9*u3*u2^2,...],[[-s-1,2]]],
[[-2048*u1^3-...],[-u3*u2,u2*u1,...],[[-s-1,1],...]]],
[[-256*u1^3+128*u3^2*u1^2+...],[...],[[-s-1,1]]],
[[],[-256*u1^3+128*u3^2*u1^2+...],[[]]]
```

#### 0.1.4 `ndbf.action_on_gfs`

`ndbf.action_on_gfs(op,v,gfs)`  
 :: computes the action of an operator *op* on  $gf^{\wedge}(s+a)$

*return* a list  
*op* a differential operator  
*gfs* a list [*g,f,s+a*]  
*v* list of variables of *f* ( $v=[v1,...,vn]$ )

- This function computes the action of a differential operator *op* on  $gf^{\wedge}(s+a)$ .
- *g* is a polynomial with variables *v1,...,vn*.
- *op* is represented by a polynomial with [*v1,...,vn,dv1,...,dvn*].
- The input list [*g,f,s+a*] represents  $gf^{\wedge}(s+a)$ .
- The result is a list [*h,f,s+c*] and it means  $hf^{\wedge}(s+c)$ , where *c* is an integer. If *op* is an operator giving b-function *b(s)*, then *c=0* for *a=1* and *h=b(s)* (global case) or *h=b(s)d(v)* (local case).

```
[...] load("nn_ndbf.rr");
[...] F=x^5-y^2*z^2$
```

```
[...] B=ndbf.bfunction(F|op=1)$
[...] ndbf.action_on_gfs(B[1],[x,y,z],[1,F,s+1]);
[-6250000000*s^13-...-2985505717194*s-245434132944,x^5-z^2*y^2,s]
[...] L=ndbf.bf_local(F,[x,0,y,0,z,1]|op=1)$
[...] ndbf.action_on_gfs(L[2],[x,y,z],[1,F,s+1]);
[(-100000*s^5-500000*s^4-990000*s^3-970000*s^2-470090*s-90090)*z^2,
x^5-z^2*y^2,s]
```

## 0.2 Computation of annihilator ideal

### 0.2.1 ndbf.ann

`ndbf.ann(f[|weight=w])` :: computes the annihilator ideal of  $f^s$  for a polynomial  $f$ .

*return* a list of differential operators

$f$  a polynomial

$w$  a list  $[w_0, w_1, \dots, w_n, w_n]$

- This function is defined in an asir-contrib package ‘`nn_ndbf.rr`’.
- This function computes the annihilator ideal of  $f^s$  for  $f$ . The output is a list of differential operators containing  $s$  in their coefficients. The differential operators are represented in the same manner as `ndbf.bf_local`.
- If an option `weight=[v1,w1,...,vn,wn]` is given, the computation is done with a weight  $(w_1, \dots, w_n)$  for  $(v_1, \dots, v_n)$ . This option is useful when  $f$  is weighted homogeneous with respect to  $(w_1, \dots, w_n)$ .

```
[...] load("nn_ndbf.rr");
[...] ndbf.ann(x*y*z*(x^3-y^2*z^2));
[(-x^4*dy^2+3*z^4*x*dz^2+12*z^3*x*dz+6*z^2*x)*dx+4*z*x^3*dz*dy^2
-z^5*dz^3-6*z^4*dz^2-6*z^3*dz,
(x^4*dy-3*z^3*y*x*dz-6*z^2*y*x)*dx-4*z*x^3*dz*dy+z^4*y*dz^2+3*z^3*y*dz,
(-x^4+3*z^2*y^2*x)*dx+(4*z*x^3-z^3*y^2)*dz,2*x*dx+3*z*dz-11*s,
-y*dy+z*dz]
```

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