

Direct sampler

Direct sampler by contiguity relations of GKZ hypergeometric systems

Version 0.1

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1 About this document

This document explains Risa/Asir functions for direct sampler by contiguity relations of GKZ hypergeometric systems

Loading the package:

```
import("gtt_ds.rr");
```

References cited in this document.

- [MT2021v2] S.Mano, N.Takayama, Algorithm for direct sampling from conditional distributions of toric models, version 2, <https://arxiv.org/abs/2110.14992>.

2 Simplicial complex to A matrix

2.1 gtt_ds.getA

`gtt_ds.getA(Levels, Facets)`

:: It returns a matrix A standings for a simplicial complex defined by *Facets* and the set of levels of each vertex *Levels*.

return Matrix A to define a GKZ system.

Levels Levels of each vertex of the simplicial complex *Facets*.

Facets The set of facets of a simplicial complex. A facet is expressed by a 0, 1 vector. 1 stands for the vertex of the position belongs to the facet and 0 stands for the vertex of the position does not belong to the facet.

- The following is an explanation of how facets are expressed by the variable *Facets*. Consider the simplicial complex of 4 vertices [123][134] (square with one diagonal line)



The facet 12 is expressed as $[1,1,0,0]$, The facet 23 is expressed as $[0,1,1,0]$, the facet 13 is expressed as $[1,0,1,0]$. The set of all facets is $[[1,1,0,0], [0,1,1,0], [1,0,1,0], [1,0,0,1], [0,0,1,1]]$.

- The variable *Levels* is a set of levels of each vertex by pressed by numbers 1, 2, 3, ... For example, if we have four vertices and *Levels* is $[2,2,3,4]$, then the vertex 1 has the levels 1,2, the vertex 2 also has the levels 1,2, the vertex 3 has the levels 1,2,3, and the vertex 4 ahs the levels 1,2,3,4.
- The rows of the matrix A is indexed by all levels of all facets. See page 4 of [MT2021v2], $\mathcal{I}_F$. Let us show an example by our four vertices simplicial complex above. Let F be the facet [13] and *Levels* $[2,2,3,4]$. Then, all levels of the facet F is $[1,0,1,0], [1,0,2,0], [1,0,3,0], [2,0,1,0], [2,0,2,0], [2,0,3,0]$. The rows of the matrix A is indexed by such index that runs over all facets.
- For a graphical model, the set of facets are the set of cliques.
- The columns of the matrix A is indexed by all possible combinations of levels. See page 4 of [MT2021v2], $calI_V$. For example, if *Levels*= $[2,4]$, then the column index is $[1,1], [1,2], [1,3], [1,4], [2,1], [2,2], [2,3], [2,4]$.
- The matrix A is a 0-1 matrix. If the row index and the column index matches on the facet F underlying the row index, the the entry of A is 1.

A is $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ For example, $[1,0],[1,1]$ matches $(1?,11, \text{replace } 0 \text{ by the wild card } ?)$

```
[2122] import("gtt_ds.rr");
[4432] gtt_ds.getA([2,2],[[1,0],[0,1]]);
[ 1 1 0 0 ]
[ 0 0 1 1 ]
[ 1 0 1 0 ]
[ 0 1 0 1 ]
```

The matrix A is $\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

```
[2122] import("gtt_ds.rr");
[4432]   gtt_ds.getA([2,3],[[1,0],[0,1]]);
[ 1 1 1 0 0 0 ]
[ 0 0 0 1 1 1 ]
[ 1 0 0 1 0 0 ]
[ 0 1 0 0 1 0 ]
[ 0 0 1 0 0 1 ]
```

The row index $[1,1,1,0]$ and the column index $[1,1,1,1]$. matches (111? and 1111). The row index $[1,1,1,0]$ and the column index $[1,1,1,2]$. matches (111? and 1112). ... The row index $[1,1,1,0]$ and the column index $[1,2,1,1]$. does not matche (111? and 1211). ... The row index $[1,1,1,0]$ and the column index $[2,2,3,4]$. does not matche (111? and 2234).

```
[2122] import("gtt_ds.rr");  
[4432] gtt_ds.getA([2,2,3,4],[[1,1,1,0],[1,0,1,1]]);  
[ 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
                                0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]  
[ 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```

                                0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
--- snip ---
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
                                0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 ]

```

Example: See pages 20, 21 of [MT2021v2].

```

[2123] import("mt_mm.rr");
[4432] gtt_ds.getA([2,2,2,2],[[1,1,1,0],[1,1,0,1]]);
[ 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 ]
[ 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 ]

```

Refer to Section 3.1 [gtt_ds.direct_sampler], page 5,

3 Direct sampler of two way contingency table

3.1 gtt_ds.direct_sampler

`gtt_ds.direct_sampler(MarginalSum, CellProb)`
 :: It returns a sample contingency table.

return A sample contingency table.

MarginalSum

Marginal sums of a two way contingency table.

CellProb A list of probabilities of each cell of the table.

- Repeating to call this function generates samples with integral entries with given marginal sums *MarginalSum*.
- The function utilizes the package `gtt_ekn3.rr` to obtain contiguity relations for the GKZ hypergeometric systems (Aomoto-Gel'fand system) for two way contingency tables.

Example: we consider 2×3 contingency tables with the cell probability $\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1 & 1 & 1 \end{pmatrix}$, the row marginal sums (5, 6), and the column marginal sums (3, 3, 5).

```
[1883] import("gtt_ds.rr");
[4447] for (I=0; I<3; I++)
    printf("%a\n-----\n", gtt_ds.direct_sampler(MarginalSum, CellProb));
[ 2 0 3 ]
[ 1 3 2 ]
-----
[ 2 2 1 ]
[ 1 1 4 ]
-----
[ 2 0 3 ]
[ 1 3 2 ]
-----
```

Todo, install a function to obtain contiguity relations for a given matrix A. Call this function with optional variable *a* (A matrix), *MarginalSum* is β . Refer to `mt_mm.rr` and latest isom-project codes.

Refer to Section 2.1 [`gtt_ds.getA`], page 2,

Index

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(Index is nonexistent)

Short Contents

1	About this document	1
2	Simplicial complex to A matrix	2
3	Direct sampler of two way contingency table.....	5
	Index	6

Table of Contents

1	About this document	1
2	Simplicial complex to A matrix	2
2.1	gtt_ds.getA	2
3	Direct sampler of two way contingency table ..	5
3.1	gtt_ds.direct_sampler	5
	Index	6

